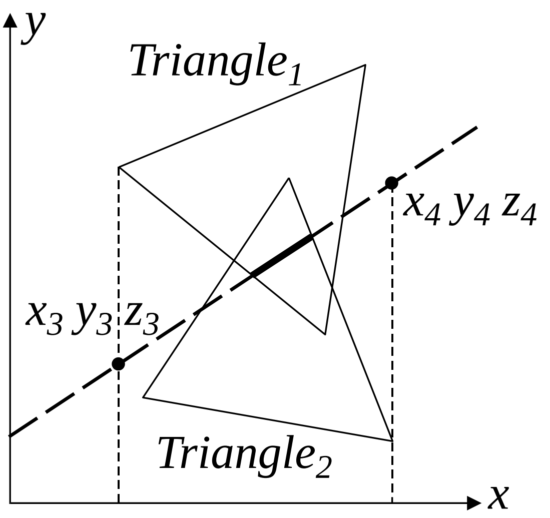
Calculation of Line of Intersection Between Two Planes Defined by 3D Triangles



Known variables:

a_1 = intersection of *Plane 1* on z axis when x and y = 0
b_1 = slope of *Plane 1* along x axis when y = 0
c_1 = slope of *Plane 1* along y axis when x = 0

a_2 = intersection of *Plane 2* on z axis when x and y = 0
b_2 = slope of *Plane 2* along x axis when y = 0
c_2 = slope of *Plane 2* along y axis when x = 0

From the equation for a 2D plane*:

$z = a + bx + cy$

(* for all planes except those perpendicular to the x, y axes when multiple z values are possible at the same x, y coordinates)

Unknown variables:

$x_3y_3z_3$ and $x_4y_4z_4$ = x, y, z coordinates on the line of intersection between *Plane 1* and *Plane 2*

(i) Initial equations:

Plane 1: $z = a_1 + b_1x + c_1y$
Plane 2: $z = a_2 + b_2x + c_2y$

(ii) Equate the z's and re-arrange to get x in terms of y and y in terms of x:

$$a_1 + b_1x + c_1y = a_2 + b_2x + c_2y$$

$$\therefore x = \frac{(a_2 - a_1) + (c_2 - c_1)y}{(b_1 - b_2)} \qquad y = \frac{(a_2 - a_1) + (b_2 - b_1)x}{(c_1 - c_2)}$$

(iii) If $b_1 - b_2 = 0$ and $c_1 - c_2 = 0$ then the planes are coplanar, and there is no intersection line.

(iv) Otherwise if $|b_1 - b_2| > |c_1 - c_2|$ start with:
x_3 = minimum x coordinate of both triangles and
x_4 = maximum x coordinate of both triangles

Then calculate y_3 and y_4 using the equations in (ii)

(v) If $|b_1 - b_2| < |c_1 - c_2|$ start with y_3 and y_4 as minimum/maximum y's and calculate x_3 and x_4

(vi) Finally, calculate where the intersection line crosses each triangle's edges.

(vii) The ends of the intersection line are defined where a crossing point of one triangle's edge is also within the other triangle's boundary. If the triangles do not overlap, no such points exist.