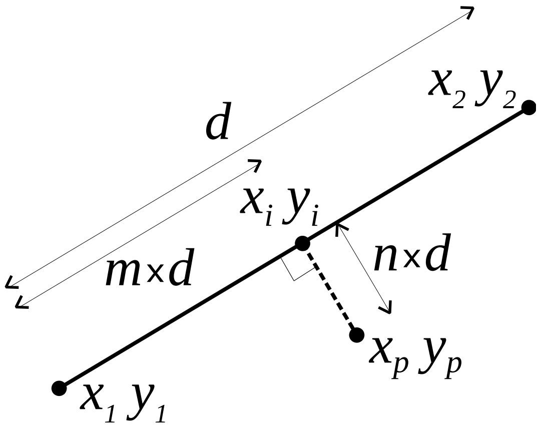
Calculation of Intersection of Line from Point Perpendicular to Line and Side of Line to Point



Known variables:

$x_1\ y_1$ and $x_2\ y_2$ (points at each end of main line)
$x_p\ y_p$ (point at start of perpendicular line)

Unknown variables:

d (distance between points $x_1\ y_1$ and $x_2\ y_2$)
$x_i\ y_i$ (point at intersection of lines)
m (ratio along main line to intersection point)
n (ratio of perpendicular line length to main line)

(i) Initial Equations:

$$x_i = x_1 + m \times (x_2 - x_1)$$
$$y_i = y_1 + m \times (y_2 - y_1)$$

$$x_i = x_p - n \times (y_2 - y_1)$$
$$y_i = y_p + n \times (x_2 - x_1)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(ii) Equate the x_i and the y_i and re-arrange so n is alone:

$$x_1 + m \times (x_2 - x_1) = x_p - n \times (y_2 - y_1)$$

$$n = \frac{x_1 + m \times (x_2 - x_1) - x_p}{-(y_2 - y_1)}$$

$$\therefore n = \frac{(x_1 - x_p) + m \times (x_2 - x_1)}{-(y_2 - y_1)}$$

$$y_1 + m \times (y_2 - y_1) = y_p + n \times (x_2 - x_1)$$

$$n = \frac{y_1 + m \times (y_2 - y_1) - y_p}{(x_2 - x_1)}$$

$$\therefore n = \frac{(y_1 - y_p) + m \times (y_2 - y_1)}{(x_2 - x_1)}$$

(iii) Equate the n and re-arrange so m is alone:

$$\frac{(x_1 - x_p) + m \times (x_2 - x_1)}{-(y_2 - y_1)} = \frac{(y_1 - y_p) + m \times (y_2 - y_1)}{(x_2 - x_1)}$$

$$\therefore \left[\left(x_1 - x_p\right) + m \times (x_2 - x_1)\right] \times (x_2 - x_1) = [(y_1 - y_p) + m \times (y_2 - y_1)] \times -(y_2 - y_1)$$

$$\therefore \left(x_1 - x_p\right) \times (x_2 - x_1) + m \times (x_2 - x_1)^2 = (y_1 - y_p) \times -(y_2 - y_1) - m \times (y_2 - y_1)^2$$

$$\therefore \left(x_1 - x_p\right) \times (x_2 - x_1) + \left(y_1 - y_p\right) \times (y_2 - y_1) = -m \times [(x_2 - x_1)^2 + (y_2 - y_1)^2]$$

(switch signs so m is positive)

$$m \times [(x_2 - x_1)^2 + (y_2 - y_1)^2] = \left(x_p - x_1\right) \times (x_2 - x_1) + \left(y_p - y_1\right) \times (y_2 - y_1)$$

$$\therefore m = \frac{\left(x_p - x_1\right) \times (x_2 - x_1) + \left(y_p - y_1\right) \times (y_2 - y_1)}{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore m = \frac{\left(x_p - x_1\right) \times (x_2 - x_1) + \left(y_p - y_1\right) \times (y_2 - y_1)}{d^2}$$

(iv) Having found m use the equations in (ii) to find n:

When $\begin{Bmatrix} x_2 - x_1 = 0 \\ y_2 - y_1 = 0 \\ x_2 - x_1 \neq 0 \text{ and } y_2 - y_1 \neq 0 \end{Bmatrix}$ use $\begin{Bmatrix} \text{left hand side} \\ \text{right hand side} \\ \text{either} \end{Bmatrix}$ equation

$x_2 - x_1 = 0$ for vertical lines and $y_2 - y_1 = 0$ for horizontal lines.

<u>Notes:</u>

Point $x_i\ y_i$ lies between points $x_1\ y_1$ and $x_2\ y_2$ on the main line when $0 < m < 1$

When $\begin{Bmatrix} n < 0 \\ n = 0 \\ n > 0 \end{Bmatrix}$ point $x_p\ y_p$ is located $\begin{Bmatrix} \text{left of} \\ \text{on} \\ \text{right of} \end{Bmatrix}$ the main line. The diagram above shows $n > 0$